These questions are just for fun, and most of them are pretty hard. Don't get discouraged if you don't know how to solve one of them! If you have any questions or want to see if you got an answer right, feel free to email me at zachary.j.winkeler.gr@dartmouth.edu.

Problem 1. Games on Chessboards

Rooks aren't the only chess piece you can play this game with. You can also play the chessboard game with a queen, a king, or a knight!

The queen can move any number of spaces up, right, or diagonally up and right. The king can move only one space up, right, or diagonally up and right.

The knight can move in an L-shape, either 1) 2 spaces up and 1 space right, 2) 2 spaces up and 1 space left, 3) 1 space up and 2 spaces right, or 4) 1 space down and 2 spaces right.

Fill in the positions on the chessboards according to which nimbers they are equivalent to.

Rook ♖

							⋆

Knight ♘

						⋆	⋆
						⋆	⋆

Queen ♕

							⋆

King ♔

							⋆

Problem 2. The Queen Game (Difficult)

In the queen version of the chessboard game, the $\mathcal{P}$-positions form two rough "lines" starting at the star. What are the slopes of their lines of best fit?

Problem 3. Kayles

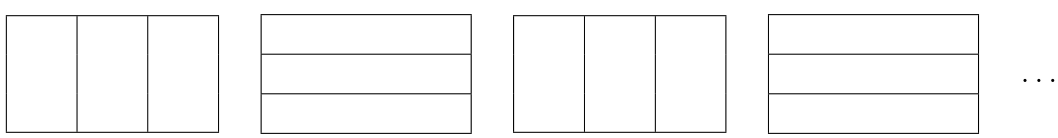
During class, we talked about the game Kayles, which is played with 13 pins lined up in a row. Players take turns knocking over either one pin or two adjacent pins; the winner is the player who knocks over the last pin.

a) Find the nim-value of Kayles.

b) Describe how to calculate the nim-value of Kayles starting with n pins. (If you know a programming language, you can write code to compute this for you!)

Problem 4. Jenga

A classic game of Jenga is played with 54 wooden blocks; the blocks are arranged so that they form an 18-block tall tower, with 3 blocks per level. Furthermore, the pattern of blocks alternates on each level, like so:



Players take turns taking one block from any level of the tower (except the row below the incomplete top level), and placing the block on the topmost tower so as to continue the pattern.

The game ends when the tower falls. Normally, this happens because people are clumsy, but our super perfect rational game-players have steady hands, and they never make mistakes. Nevertheless, after enough moves, the tower will be in a state where it is impossible to remove a block without causing the whole tower to fall. For the sake of having an interesting game, assume that our perfect players give up when they reach this point.

With these rules, Jenga is actually an impartial game! Which player wins? Which positions are $\mathcal{N}$- and $\mathcal{P}$-positions? Can you figure out the nim-values for Jenga positions? What should your winning strategy be?

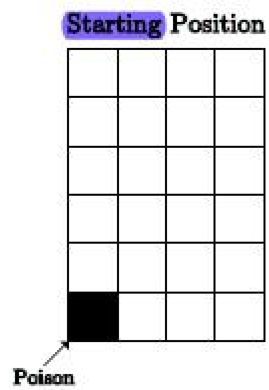
Problem 5. Misère Notakto

As we found out, Notakto with the normal play convention is quite boring. Figure out a strategy for Notakto under *misère play*. (Recall that this means that the first person to make 3-in-a-row is the *loser*, instead of the winner.)

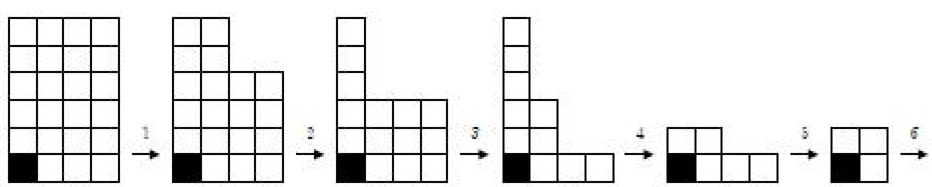
Problem 6. Chomp

Chomp is a game about a poisoned bar of chocolate.

Exercise 2.5.11. The game of *Chomp*. The starting position consists of an $n \times m$ grid with one poison square in the lower-left corner. For example, here is the 6×4 case:



One makes a move by eating any square along with all others above or to the right of it. The first 5 moves of a game in progress between two beginners might look like this:



The player who is forced to eat the poisoned square loses.

a) What kind of game is Chomp? (impartial or partizan, normal or misère)

b) Which player has a winning strategy when the candy bar is 2×2? What about when the bar is 3×2?

c) For a general $n \times m$ candy bar, when does either player have a winning strategy?

Problem 7. Octal games

Octal games generalize games like Nim that are played with piles of tokens. Like Nim, a player wins when they take the last token from the last pile. An octal game is specified by a sequence of digits

$$0.d_1 d_2 d_3 \ldots$$

The digit d_n specifies whether or not the player is allowed to take n tokens from a pile, and how many piles they are allowed to leave after taking the tokens. The digit d_n is the sum of

- 1 if leaving zero piles is permitted,
- 2 if leaving one pile is permitted, and
- 4 if the player may split a pile into two non-empty piles after taking some tokens.

For example, in the octal game 0.305, a player may either

- take 1 token from a pile,
- take the last 3 tokens from a pile with exactly 3 tokens, or
- take 3 tokens from a pile with at least 5 tokens, and then split that pile into two smaller piles.

Questions:

a) What octal game is Nim?

b) Find an octal game that is equivalent to the n-pin generalization of Kayles from Problem 3.

Problem 8. More Impartial Games

Find more impartial games out in the wild and analyze them! Some examples of games I've found that are impartial:

- Jenga
- Don't Break the Ice
- Quarto