

Will the bird get home? and other stories in random walks

Rainstorm 2020 Teaching Notes

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1 Basics

In this section, we record some basic information.

Number of students: Up to 20.

Age group: Any.

Time: 55 minutes.

Goals:

- Introduce students to the idea of random walks.
- Explore the questions of what such walks would look like across different surfaces.
- Pique students' interest in mathematical phenomena.
- Have some fun!

2 Outline of plan

(0-5) Let students enter, get settled in.

(5-10) Introduction to random walks on the number line \mathbb{Z} .

(10-15) Answer the question of whether such a random walk will ever return home.

(15-25) Discuss how the question changes when we apply it to a 2D lattice, \mathbb{Z}^2 . What is the expected number of times that we return to the origin?

25-30) The obvious strategy doesn't work!

(30-40) Apply Stirling's formula and p-series divergence tests to find the answer to the \mathbb{Z}^2 case.

(40-50) Discuss the obvious modification in the 3D case, \mathbb{Z}^3 . What does it imply?

(50-55) Closing remarks. Hope they had fun!

Other notes:

- Plan is to write out information on Zoom whiteboard, so that information will be presented as the class moves forwards.
- We will provide written-up notes at the end of class (via email) so that they can review and look back at it when class is over.