Rainstorm 2020

The Science of Chaos and Fractals: an Introduction

Number of students: no lower or upper limit
Age group: 9th grade and up
Time: 55 minutes
Prerequisites: comfort with functions and systems of equations expected. Calculus and programming experience not required, but helpful

Learning Objectives:

- Students should get a big picture overview of the mathematics and the physical implications of chaotic behavior, and some of the historical context in which these advancements in understanding were made
- Students should be able to define chaos (very sensitive dependence on initial conditions) in intuitive language, and point out some chaotic systems in the real world
- Students should understand (at a high level) the relationship between chaos and fractals, and how certain processes generate self-similar structures, both in nature and mathematics
- Students who have some programming experience should gain the tools to do some simple numerical experiments that demonstrate counterintuitive results

Plan:

20 minutes

- Historical context and outstanding problems in physics in mid 20th century
- First inklings of chaotic systems in numerical experiments: Lorenz system (atmospheric dynamics), Feigenbaum map and period doubling (population dynamics)
- Observation of chaos in physical systems (double pendulum, turbulence, dripping tap)

20 minutes

- What is chaos, really? A general definition and illustration of this definition
- How does this break the standard 'physics paradigm?'
- Relation between chaos and fractals - fine scale repeating structure in phase space
- How are fractals generated? Examples in Newton's method, Mandelbrot set
- Observation of fractals in nature (trees branching, cauliflower heads)

15 minutes

- John Conway's game of life, Wolfram 'Rule 30' as illustration of simple rules leading to complex behavior
- Tie back into chaos theory with the takeaway: simple rules can generate complex patterns (and they often do)! Observing the resultant behavior and deducing the rules behind it is what science is all about!
- Questions

Throughout the lesson there will be slides that illustrate the points being made, and demonstrations, particularly with regard to the numerical aspect of things, which hopefully make the topics more accessible, as students can see the small number of rules that are required to produce unpredictable and complex behavior.